# Worcester County Mathematics League 

WOCOMAL Varsity Meet \#3

# Coaches' Booklet 

February 7, 2007

## Round 1: Similarity and Pythagorean Theorem

## ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM.

1. What is the length of $\overline{B C}$ in right $\triangle A B C$ if $\overline{A D} \perp \overline{B C}$ ? $\Varangle B A C=90^{\circ} ; \overline{A D}=4 ; \overline{B D}=6$ 。

2. In right triangle ABC shown below, $\overline{A C}=4$ and $\overline{B C}=3 . C D E F$ is a square. What is the length of $\overline{E F}$ ?

3. Find the length of $A E$ in the rectangle below if $B E=4, C E=5$, and $D E=8$.


ANSWERS
(1 pt.)

1. $\qquad$
(2 pts.)
2. $\qquad$
(3 pts.)
3. $\qquad$
Bartlett, Bromfield, and Doherty

## Round 2: Algebra 1 (open)

## ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM.

1. If $x+2 y=11$ and $3 x+y=13$, then find the value of $(x+y)$.
2. The ages of Sean and Bryan now total 49 years. Bryan's age now is twice the age that Sean's was when Bryan's age was the same as Sean's is now. How many years old is Bryan?
3. If a and b are roots of $7 x^{2}-2002 x+182=0$, find the value of $\frac{1}{a}+\frac{1}{b}$.

ANSWERS
(1 pt.)

1. $\qquad$
(2 pts.)
2. $\qquad$
(3 pts.)
3. $\qquad$

Bancroft, Burncoat, and Quaboag

## Round 3: Functions (NO CALCULATORS)

## ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM.

1. If $f(x)=2 x^{2}-3 x+1$, then $f(x-3)=$ ? Express your answer as a polynomial in x .
2. Let P and Q be polynomials such that $\mathrm{P}(\mathrm{x})$ and $\mathrm{Q}(\mathrm{P}(\mathrm{Q}(\mathrm{x})))$ have the same roots. If the degree of $P$ is 7, what is the degree of Q ?
3. For all real numbers x , the function $f(x)$ satisfies $2 f(x)+f(1-x)=x^{2}$.

Find $f(5)$.

## ANSWERS

(1 pt.)

1. $\qquad$
(2 pts.)
2. $\qquad$
(3 pts.)
3. $\qquad$

St. John's, Tahanto, and Tantasqua

## Round 4: Combinatorics

## ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM.

1. The Yurek family is made up of eight people. One night, they gather for a family dinner at their uniformly circular, eight place dinner table. In how many ways can the Yurek family sit if the two parents must sit together?
2. Using all the letters, how many distinct new "words" can be produced from the letters of MATHEMATICS? Note: "SCITAMEHTAM" is considered a "word".
3. How many subsets of a 25 -element set have more than three elements?

ANSWERS
(1 pt.)

1. $\qquad$
(2 pts.)
2. $\qquad$
(3 pts.)
3. $\qquad$

Doherty, Shepherd Hill, and Worcester Academy

## Round 5: Analytic Geometry (NO CALCULATORS)

1. The points $\mathrm{Q}=(9,14)$ and $\mathrm{R}=(\mathrm{a}, \mathrm{b})$ are symmetric with respect to the point $\mathrm{P}=(5,3)$. What are the coordinates of point R ?
2. Find the area of a triangle with vertices $(-6,9),(4,3)$, and $(2,-2)$.
3. Find the coordinates of the point on the circle $(x+1)^{2}+(y-5)^{2}=10$ that is closest to the line $y=3 x+20$.
ANSWERS
(1 pt.)
4. $\qquad$
(2 pts.)
5. $\qquad$ square. units
(3 pts.)
6. $\qquad$

Bromfield, Notre Dame, and Shrewsbury

## ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM AND WRITTEN ON <br> THE SEPARATE TEAM ANSWER SHEET. <br> (2 points each)

1. A train traveling between 2 towns arrives 9 minutes late when traveling at 36 mph . When it travels at 27 mph the train arrives 39 minutes late. How many miles are there between the two towns?
2. Given $x^{2}-34 x+A=(x-B)^{2}$, find $B \sqrt{A}$ if $\sqrt{A}$ is the largest of its solutions.
3. If $x^{2}-3 x-21=y$ and $x^{2}-4 x-18=y$, what is the product of $(x-5)(x-7)(x+2)(x+3)$ in terms of y ?
4. There are two candy jars, each of which contains the same number of pieces of candy. Bert takes 25 pieces of candy from the first jar and gives the rest to Karla, while Karla takes 17 pieces of candy from the second jar and gives the rest to Bert. When they are done, Karla discovers that Bert has more pieces of candy than she does. How many more pieces of candy does Bert have?
5. Let f be a function with $\frac{f(x) f(y)-f(x y)}{3}=x+y+2$. List all possible values for $f(36)$.
6. How many counting numbers have four distinct nonzero digits such that the sum of the four digits is 12 ?
7. $A B E D, D F G E, B G H C$, and $A C H F$ are all similar. Find the ratio, $\frac{A B}{A D}$.

8. A square in the plane has a pair of opposite vertices at the points $(2,4)$ and $(-2,2)$. If the points $(a, b)$ and $(c, d)$ are the other two vertices, determine the value of $a+b+c+d$.
9. If $x y=3, x z=4$, and $y z=6$, Find the value of $x^{2}+y^{2}+z^{2}$.

Assabet Valley, Auburn, Leicester, Nashoba, Worcester Academy

February 7, 2007
Round 1: Similarity and Pythagorean Theorem

| 1. | (1 pt.) | $\frac{26}{3}, 8 \frac{2}{3}, 8 . \overline{6}$ |
| :--- | :--- | :--- |
| 2. | (2 pts.) | $\frac{12}{7}, 1 \frac{5}{7}, 1 . \overline{714285}$ |
| 3. | (3 pts.) | $\sqrt{55}$ |
| Round 2: Algebra 1-open |  |  |
| 1. | (1 pt.) | 7 |
| 2. | (2 pts.) | 28 |
| 3. | (3 pts.) | 11 |
| Round 3: Functions |  |  |

1. (1 pt.) $2 x^{2}-15 x+28$
2. (2 pts.) 1
3. (3 pts.) $\frac{34}{3}, 11 \frac{1}{3}, 11 . \overline{3}$

WOCOMAL Varsity Meet ANSWERS

## Round 4: Combinatorics

1. (1 pt.) 1440
2. (2 pts.) 4,989,600
3. (3 pts.) 33,551,806

Round 5: Analytic Geometry

1. (1 pt.) (1,-8)
2. (2 pts.)

31
3. (3 pts.)
$(-4,6)$

February 7, 2007
TEAM ROUND (2 pts. Each)

## 1. 54

2. 289
3. $y^{2}+8 y-33$ or $(\mathbf{y}+\mathbf{1 1})(\mathbf{y}-\mathbf{3})$
4. 16
5. 39
6. 48
7. $\frac{\sqrt{2+2 \sqrt{5}}}{2}$
8. 6
9. $\frac{29}{2}, 14 \frac{1}{2}, 14.5$

February 7, 2007
WOCOMAL Varsity Meet
TEAM ROUND
ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM AND ON THIS SEPARATE TEAM ANSWER SHEET. (2 points each)

1. $\qquad$
2. $\qquad$
3. $\qquad$
4. $\qquad$
5. $\qquad$
6. $\qquad$
7. $\qquad$
8. $\qquad$
9. 

February 7, 2007

TEAM ROUND

WOCOMAL Varsity Meet

School: $\qquad$
Team \#: $\qquad$

## Team Members:

1. 
2. 
3. 
4. 
5. 
